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NAVY DEPARTMENT

Report on
The Velocity of Sound in Sea Water

NAVAL RESEARCH LABORATORY
ANACOSTIA STATION
WASHINGTON, D. C.

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ABSTRACT

The purpose of this report is to present the latest available data on the velocity of sound in sea water as modified by changes in temperature, salinity, and depth. These are given in both metric and English units in a series of tables, curves, and equations for depths to 1300 feet.

The basic data have been compiled from the published reports of competent authorities who calculated the velocities from fundamental physical equations whose constants have been experimentally determined with precision. They have been checked with numerous direct measurements, and agree within an experimental error estimated to be less than 0.2%. The soundness of the theoretical basis for the calculations and the consistency of the results indicate that the calculated values are accurate to within less than 0.05% at temperatures below 70° F.

A bibliography of source materials is included.

TABLE OF CONTENTS

The Velocity of Sound in Sea Water

INTRODUCTION

1. Experimental determination of the velocity of sound in sea water over wide ranges of temperature and salinities presents many difficulties. Consequently several computed tables have been published based on fundamental physical theory and data. These tables are more accurate and comprehensive than direct experimental determinations.

2. The purpose of this report is to present some of the latest available data in a form convenient for use.

DEFINITION OF SALINITY

3. The salinity is a measure of the salt content of the water in parts per thousand by weight and is written S = ppt. or 0/00.

4. The salinity of the open sea water may range from 30 to 40 ppt. "Standard" salinity is conventionally chosen as 35 ppt.

CALCULATION OF THE VELOCITY OF SOUND IN WATER

5. The velocity is given by the expression

$$V = \sqrt{\frac{dp}{d\rho}} \qquad (1)$$

where p is the pressure and ρ is the density. For convenience in calculation from well established physical data the formula used is

$$V = \sqrt{\frac{\gamma}{\rho C}} \qquad (2)$$

where γ is the ratio of the specific heats at constant volume and constant pressure and C is the compressibility. The numerical values of these quantities depend on the temperature, salinity, and depth or pressure. The detailed formulae and the necessary corrections are explained fully in the references to the original data.

CALCULATED VELOCITY DATA

6. The first comprehensive tables were published by Heck and Service in 1924. They checked their tables by wire soundings, and concluded that the calculated values were probably more accurate than the experimental determinations. In 1927, the British Admiraly published an excellent set of tables for both pure and sea water that were the most extensive and accurate at that time. In 1939, Kuwahara brought out a new set of velocity tables with

added refinements and [illegible] In 1939 the British Admiralty published a revised edition of these tables which agree very closely with Kuwahara's. Wood, in his "Textbook of Sound" (1932), gives an empirical equation for the velocity in sea water at "zero" depth.

7. Table 1 shows a comparison of velocities under different conditions as given by the above authorities.

8. Since the tables of Kuwahara are believed to be the most accurate available at this time, they have been used in the preparation of the sea water curves and equations given in later sections of this report. For fresh water and slightly saline water, the values used are those of the British Admiralty tables of 1939.

VELOCITY TABLES

9. Because the velocity tables of Kuwahara may not readily be available, a part of his data is given as Table 2. For depths less than 400 meters, a pressure correction of 0.0181 meter per second per meter of depth should be added. For greater depths, the pressure correction is not so simple, and reference should be made to the original source.

VELOCITY CURVES

10. Since the effect of pressure is not great, and a correction may easily be made for depths not exceeding about 400 meters or 1300 feet, the velocity curves in this report have been drawn for zero depth.

11. Plate 1 gives the velocity in meters per second in ordinary sea water for Centigrade temperatures. These curves are as precise as the data from which they were drawn – that is, to the nearest 0.1 meter per second.

12. Plate 2 shows in metric units the temperature coefficient of velocity – that is, the change in velocity in meters per second for a change in the temperature of one degree Centigrade. It is to be noted that this coefficient is by no means constant, decreasing from about 4.7 to about 2.0, so that it cannot be used to calculate accurately the velocity at one temperature from a knowledge of the velocity of another temperature, unless the two temperatures are quite close together. Although this curve was drawn for S = 35 ppt., little error will be introduced by using it for any salinity between 30 ppt. and 40 ppt. In case greater accuracy is required, the equations given in Paragraphs 17 to 22 inclusive, may be used.

13. Plate 3 shows approximately the salinity coefficient of velocity – that is, the change in velocity in meters per second for a change in the salinity of one ppt. This coefficient is nearly independent of the salinity, so that for a fixed temperature, the velocity at one salinity may be calculated from a knowledge of the

velocity at some other value of salinity. For greater accuracy, the equations given in Paragraphs 17 to 22 inclusive may be used.

14. Plate 4 gives a summary on one sheet of the data of the preceding plates with some loss of precision due to the smaller scale.

15. Plate 5 shows the velocity of sound in pure and nearly pure water, in metric units, with a precision of about 0.5 meter per second.

16. Plates 6 to 10 give the same data as plates 1 to 5, expressed in English units of feet and Fahrenheit degrees.

VELOCITY EQUATIONS

17. Two empirical equations have been set up and the constants calculated to fit the data given in the tables and curves with the precision specified. In order to preserve the accuracy of the equations, the variables should not fall outside the ranges indicated. The following symbols will be employed.

V_m = velocity of sound in sea water, in meters per second

V_f = velocity of sound in sea water, in feet per second

S = salinity in ppt., between 30 and 40 ppt.

y_m = depth in meters below sea surface; between 0 and 400 meters

y_f = depth in feet below sea surface; between 0 and 1300 ft.

t_c = water temperature in °C; between 0 and 32°C.

t_f = water temperature in °F; between 32 and 90°F.

18. For a first approximation, a quadratic function of the temperature was assumed. Using the Method of Least Squares, an expression for the velocity in meters per second as a function of the Centigrade temperature was obtained for zero depth and standard salinity of 35 ppt. Then this expression was modified to represent the velocity in feet per second for Fahrenheit temperatures. These equations are:

$$V_m = 1445.75 + 4.596\, t_c - 0.04235\, t_c^2 \qquad (3)$$

and

$$V_f = 44[illegible] + 10.[illegible]\, t_f - 0.[illegible]\, t_f^2 \qquad (4)$$

Values of velocity [illegible] (3) agree with the values given by Kuwahara [illegible] error [illegible] m/sec, or [illegible], and an average error of [illegible], or [illegible]. However, values for the

difference in velocity per degree temperature difference calculated with the aid of equation (3) may be in error by 1% to 10%. Equation (4) is of the same percentage precision as equation (3).

19. Greater concordance with the data may be obtained from a cubic equation. Using the Method of Least Squares, an equation for the velocity in meters per second as a function of the Centigrade temperature was obtained for zero depth and standard salinity of 35 ppt. Then this equation was modified to represent the velocity in feet per second for Fahrenheit temperatures. These equations are:

$$V_m = 1445.44 + 4.6684\, t_c - 0.056{,}527\, t_c^{\,2} + 0.0003151\, t_c^{\,3} \qquad (5)$$

and

$$V_f = 4742.2 + 8.509\,(t_f - 32) - 0.057{,}239\,(t_f - 32)^2 + 0.0001773\,(t_f - 32)^3 \qquad (6)$$

Values of velocity computed with the aid of equation (5) agree with the values given by Kuwahara to within a maximum error of 0.1 m/sec, or 0.007%, and an average error of 0.04 m/sec, or 0.003%. Moreover, it is believed that the values for the difference in velocity per degree difference in temperature, as calculated from equation (5), are precise to 0.01 m/sec, or 0.5% or better. Equation (6) is of the same percentage precision as equation (5).

20. Having obtained the above cubic expressions for the velocity in terms of the temperature at standard salinity and zero depth, other terms for the salinity and pressure effects were added, yielding

$$V_m = 1445.44 + 4.6684\, t_c - 0.056527\, t_c^{\,2} + 0.0003151\, t_c^{\,3} + [1.304 - 0.0106\, t_c + 5.7 \times 10^{-8} t_c^{\,4}][(S-35) + 5.2 \times 10^{-5}\, t_c\,(S-35)^2] + 0.0181\, y_m \qquad (7)$$

and

$$V_f = 4742.2 + 8.509\,(t_f - 32) - 0.057{,}239\,(t_f-32)^2 + 0.0001773\,(t_f - 32)^3 + [4.278 - 0.0193(t_f-32) + 1.78 \times 10^{-8}(t_f-32)^4][(S-35) + 2.9 \times 10^{-5}\,(t_f-32)\,(S-35)^2] + 0.0181\, y_f \qquad (8)$$

21. The precision of equations (7) and (8) is not as great as the precision of equations (5) and (6). However, in about 100 well-distributed trials over a temperature range of 0° to 30°C, over a salinity range of 30 to 40 ppt. and over a depth range of 0 to 400 meters, the velocities computed from equation (7) agreed with the velocities given by Kuwahara's tables to 0.1 meter per second or better. The larger discrepancies were found at the extremes of temperature, salinity and depth. The percentage precision of equation (8) is the same as that of equation (7).

22. To obtain the rate of change of velocity with respect to

temperature, salinity, or depth, equations (7) and (8) may be differentiated, obtaining:

$$\frac{dV_m}{dt_c} = 4.6684 - 0.113{,}054\, t_c + 0.0009453\, t_c^{\,2} + [-0.0106 + 0.23 \times 10^{-6}\, t_c^{\,3}]\ (S-35) + [6.8 \times 10^{-5} - 0.110 \times 10^{-5}\, t_c + 1.5 \times 10^{-11}\, t_c^{\,4}]\ (S-35)^2$$

(constant salinity and depth) (9)

$$\frac{dV_m}{ds} = [1.304 - 0.0106\, t_c + 5.7 \times 10^{-8} t_c^{\,4}][1 + 1.04 \times 10^{-4} t_c (S-35)]$$

(constant temperature and depth) (10)

$$\frac{dV_m}{dy_m} = 0.0181$$

(constant temperature and salinity) (11)

$$\frac{dV_f}{dt_f} = 8.509 - 0.114{,}478(t_f - 32) + 0.0005318)\ (t_f - 32)^2 + [-0.0193 + 7.1 \times 10^{-8}\ (t_f-32)^3]\ (S-35) + [12.4 \times 10^{-5} - 0.112 \times 10^{-5}(t_f-32) + 25.8 \times 10^{-13}(t_f-32)^4]\ (S-35)^2$$

(12)

(constant salinity and depth)

$$\frac{dV_f}{ds} = [4.278 - 0.0193\ (t_f-32) + 1.78 \times 10^{-8}\ (t_f-32)^4][1 + 5.8 \times 10^{-5}(t_f - 32)\ (S-35)]$$

(13)

(constant temperature and depth)

$$\frac{dV_f}{dy_f} = 0.0181$$

(constant temperature and salinity) (14)

EXAMPLES IN THE USE OF THE GRAPHS AND EQUATIONS

Example 1

23. To find the approximate velocity in meters per second for a temperature of 16.5°C, a salinity of 34.2 ppt. and a depth of zero.

24. At 16.5°C, the curves on Plate 1 show V = 1506.2 for S=33, and V = 1508.6 for S=35. Since equation (7) shows that the salinity correction varies approximately linearly with salinity, we have that the velocity for S = 34.2 is

$$V = 1506.2 + \frac{34.2 - 33.0}{35.0 - 33.0}\ (1508.6 - 1506.2)$$

$$= 1507.6, \text{ meters per second.}$$

Example 2

25. Using the curves on Plate 4, determine the velocity in meters per second for the same data as in Example 1.

26. At 16.5°C, Curve 1 of Plate 4 shows a velocity of 1508.5 for S=35.0. At 16.5°C, Curve 3 shows a salinity coefficient of 1.132. Hence, the desired velocity is

$$V = 1508.5 - (35.0 - 34.2) \times 1.137 = 1507.6 \text{ meters}$$

per second.

Example 3

27. For the same temperature and salinity data as in Example 1, but for a depth of 275 meters, find the velocity.

28. Since the velocity correction is 0.0181 m/sec per meter of depth (see equation 7), the desired velocity is

$$V = 1507.6 + 0.0181 \times 175 = 1510.8 \text{ meters per second.}$$

Example 4

29. At a temperature of 8.8°C, and a salinity of 35 ppt., determine the temperature rate of variation of velocity.

30. From Plate 2, the answer is found to be 3.741 meters per degree Centigrade. From equation (9) the calculated value is 3.747 meters per second per degree Centigrade.

ADDITIONAL DATA

31. The Naval Research Laboratory is collecting data in this field and would appreciate contributions from any source, theoretical or experimental, or suggestions as to convenient forms of presentation.

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5. Velocity of Sound in Sea Water, by E. B. Stephenson, Physical Review, vol. 21, pp. 181-185, February, 1923.

6. Recent Acoustic Work of the U. S. Coast and Geodetic Survey, by P. A. Smith, Field Engineers Bulletin No. 8, December, 1934, pp. 60 - 75.

7. Velocity and Ray Paths of Sound Waves in Sea Water, by O. W. Swainson, Field Engineers Bulletin No. 10, December, 1936.

Appendix

The following table gives approximate values for the velocity of sound in several media. These values have been taken from tables published in handbooks and textbooks, and are believed to be reliable within 1% to 5%.

Temperature = 20° C. = 68° F., unless otherwise noted.

Material	Meters per Second	Feet per Second
Aluminum	5100	16,700
Brass	3500	11,500
Copper	3560	11,700
Iron, cast	4700	15,400
Lead	1230	4,030
Nickel	4970	16,300
Silver	2610	8,560
Steel	5000	16,400
Tin	2500	8,200
Zinc	3700	12,100
Brick	3650	12,000
Cork	430–530	1400–1740
Glass	5000–6000	16,400–19,700
Ice, 0° C.	3200	10,500
Paraffin, 15° C.	1300	4,260
Quartz	5500	18,000
Rubber	30–70	98–230
Wood	1000–4700	3300–15,400
Alcohol, 12.5° C.	1240	4,070
Castor oil	1555	5,100
Benzol, 17° C.	1170	3,840
Mercury	1410	4,625
Air (dry, 760 mm.)	344	1,128
Carbon dioxide, 0° C	258	846
Hydrogen, 0° C.	1270	4,170
Methane, 0° C.	432	1,417
Oxygen, 0° C.	317	1,040
Steam, 100° C	405	1,328
Air, 0°C. (dry, CO_2 free, 760 mm)	331.4	1,087.3

TABLE 1

VELOCITY OF SOUND IN SEA WATER

(From various tables and formulas)

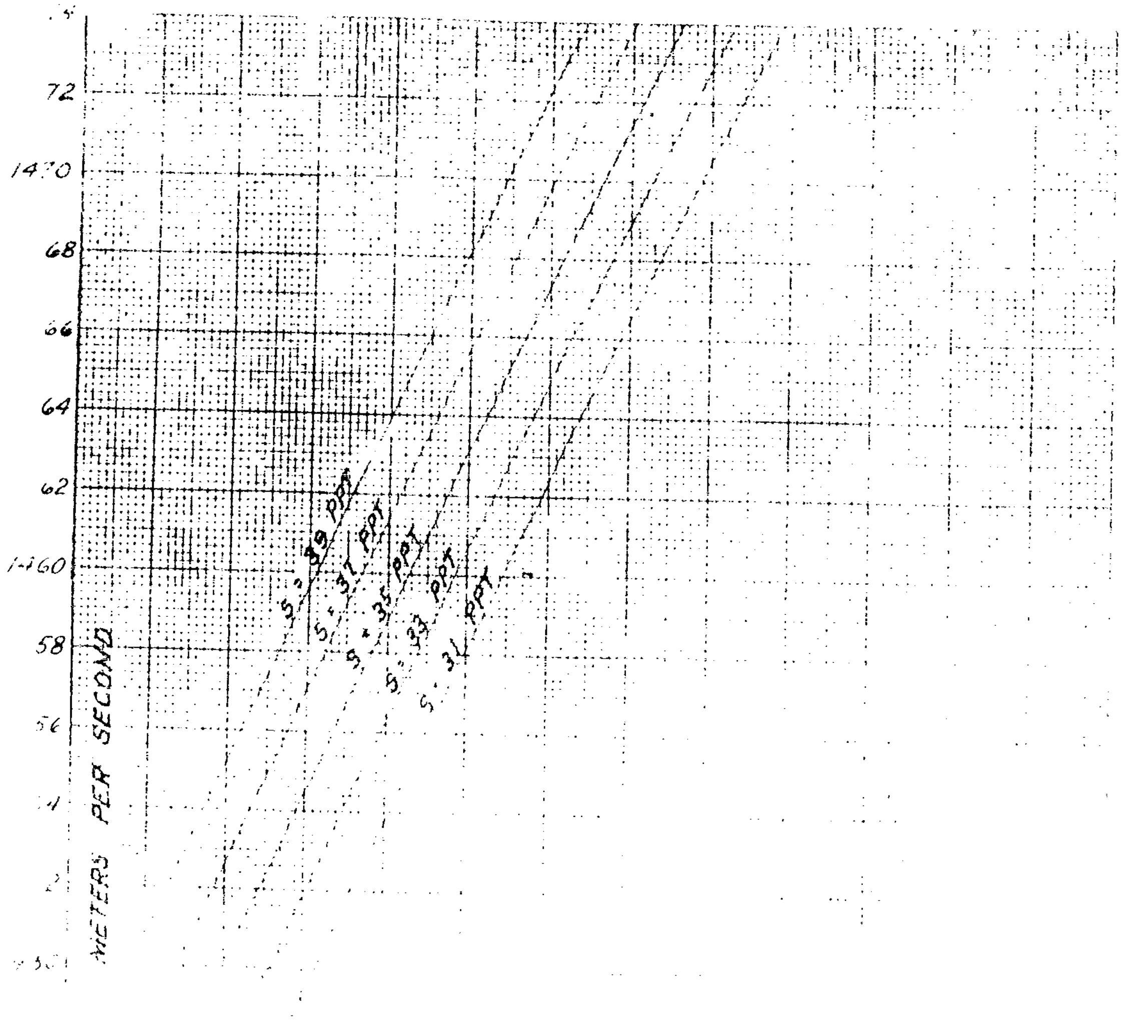
Depth in Meters	Temp. °C.	Sal. ppt.	Meters per Second				
			Heck & Service	Wood	Br.Adm. 1927	Br.Adm. 1939	Kuwahara
0	0	31	1445	1445	1440.3	1440.2	1440.3
"	10	"	1482	1484	1481.9	1481.9	1482.0
"	20	"	1508	1515	1514.3	1514.3	1514.3
"	30	"	—	1538	1539.0	1538.9	1539.1
0	0	35	1450	1450	1445.3	1445.4	1445.5
"	10	"	1489	1488	1486.6	1486.7	1486.8
"	20	"	1514	1519	1518.6	1518.7	1518.7
"	30	"	—	1543	1543.0	1543.1	1543.2
400	0	35	1454	—	1452.6	1452.7	1452.8
"	10	"	1492	—	1493.8	1493.9	1494.1
"	20	"	1518	—	1525.8	1525.9	1525.9
"	30	"	—	—	1550.3	1550.4	1550.6
3000	0	31	1490	—	1494.7	1494.6	1494.4
"	"	35	1498	—	1499.7	1499.8	1499.8

TABLE [illegible]

Velocity of Sound in Sea Water – Degrees C

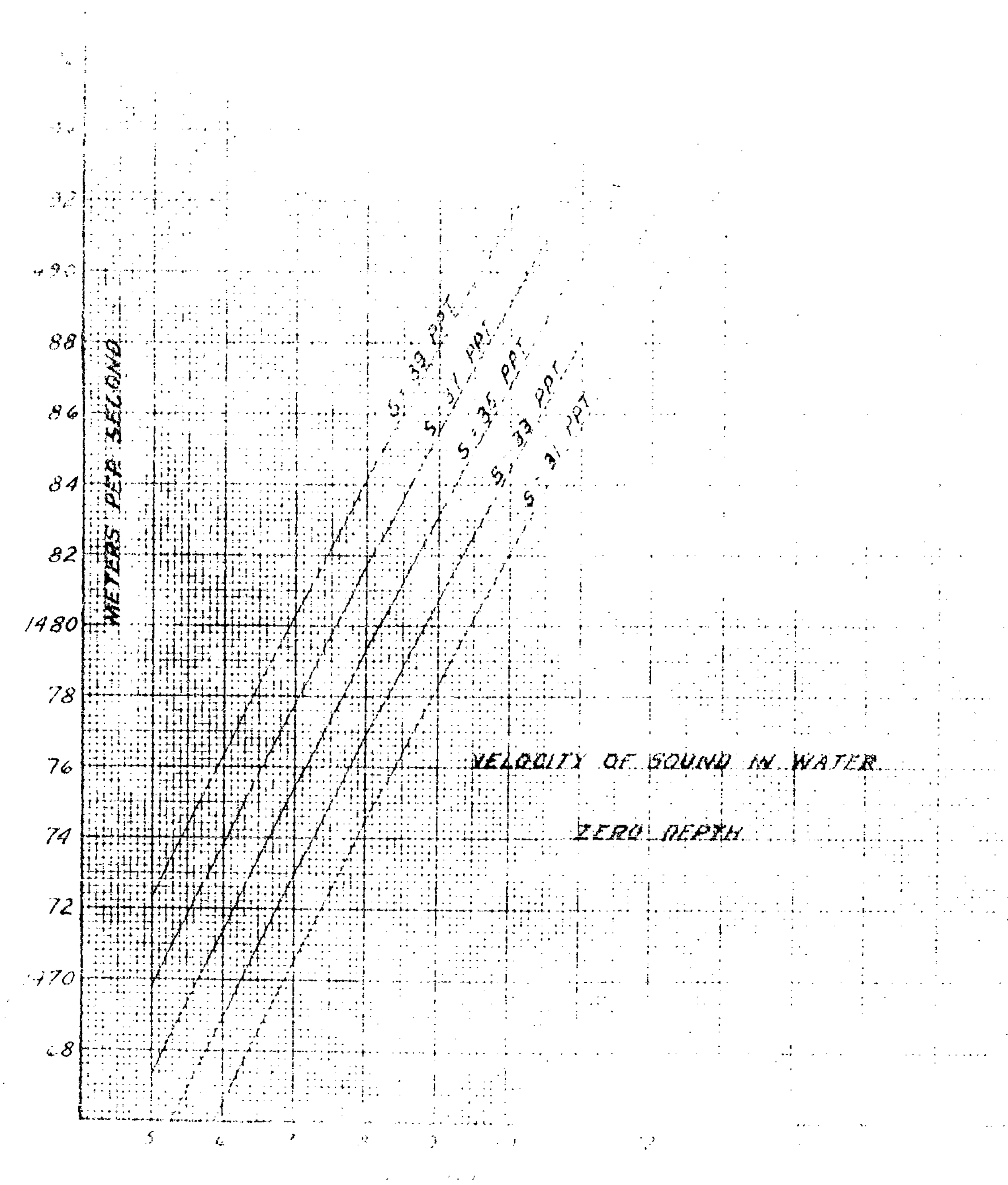
(From Kuwahara)

Degrees Cent.	Meters Per Second				
	S=31 ppt	S=33 ppt	S=35 ppt	S=37 ppt	S=39 ppt
0	1440.3	1442.9	1445.6	1448.1	1450.7
1	44.8	47.4	50.0	52.6	55.2
2	49.4	51.9	54.5	57.1	59.6
3	53.8	56.4	58.9	61.4	64.0
4	58.1	60.6	63.1	65.6	68.1
5	1462.3	1464.8	1467.3	1469.8	1472.3
6	66.5	68.9	71.4	73.9	76.3
7	70.5	73.0	75.4	77.9	80.3
8	74.5	76.9	79.3	81.7	84.2
9	78.3	80.7	83.1	85.5	87.9
10	1482.0	1484.4	1486.8	1489.2	1491.6
11	85.7	88.0	90.4	92.8	95.1
12	89.2	91.6	93.9	96.3	98.6
13	92.7	95.0	97.3	99.6	1502.0
14	96.0	98.3	1500.6	1502.9	05.2
15	1499.3	1501.6	1503.9	1506.2	1508.5
16	1502.5	04.7	07.0	09.3	11.5
17	05.6	07.9	10.1	12.3	14.6
18	08.6	10.8	13.0	15.2	17.5
19	11.5	13.7	15.9	18.1	20.3
20	1514.3	1516.5	1518.7	1520.9	1523.1
21	17.2	19.3	21.5	23.7	25.9
22	19.8	22.0	24.1	26.3	28.4
23	22.4	24.6	26.7	28.8	31.0
24	25.0	27.1	29.2	31.3	33.5
25	1527.5	1529.6	1531.7	1533.8	1535.9
26	29.9	32.0	34.1	36.2	38.3
27	32.3	34.3	36.4	38.5	40.6
28	34.6	36.6	38.7	40.8	42.9
29	36.9	39.0	41.0	43.1	45.1
30	1539.1	1541.2	1543.2	1545.3	1547.3

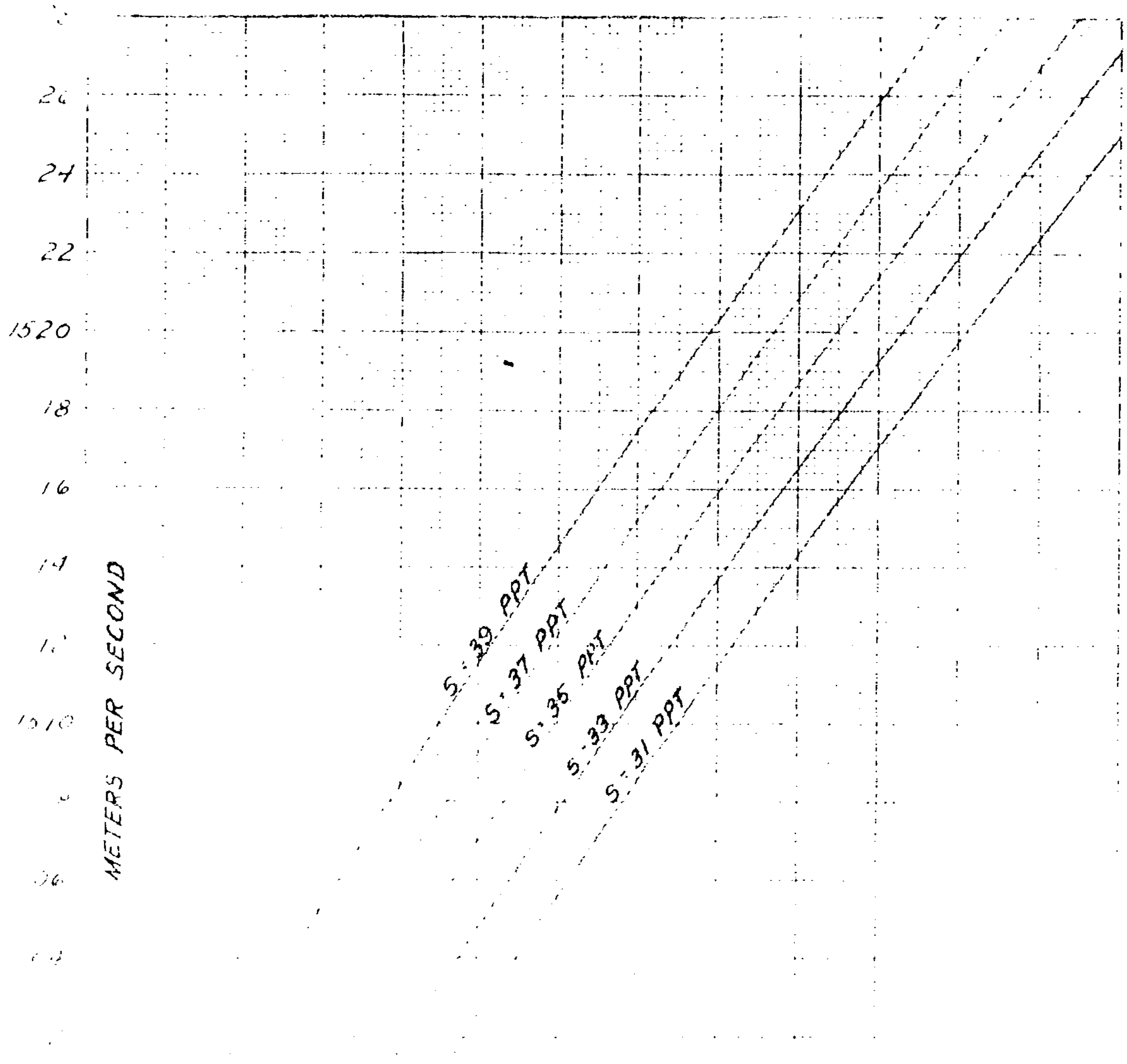


VELOCITY OF SOUND IN WATER

ZERO DEPTH

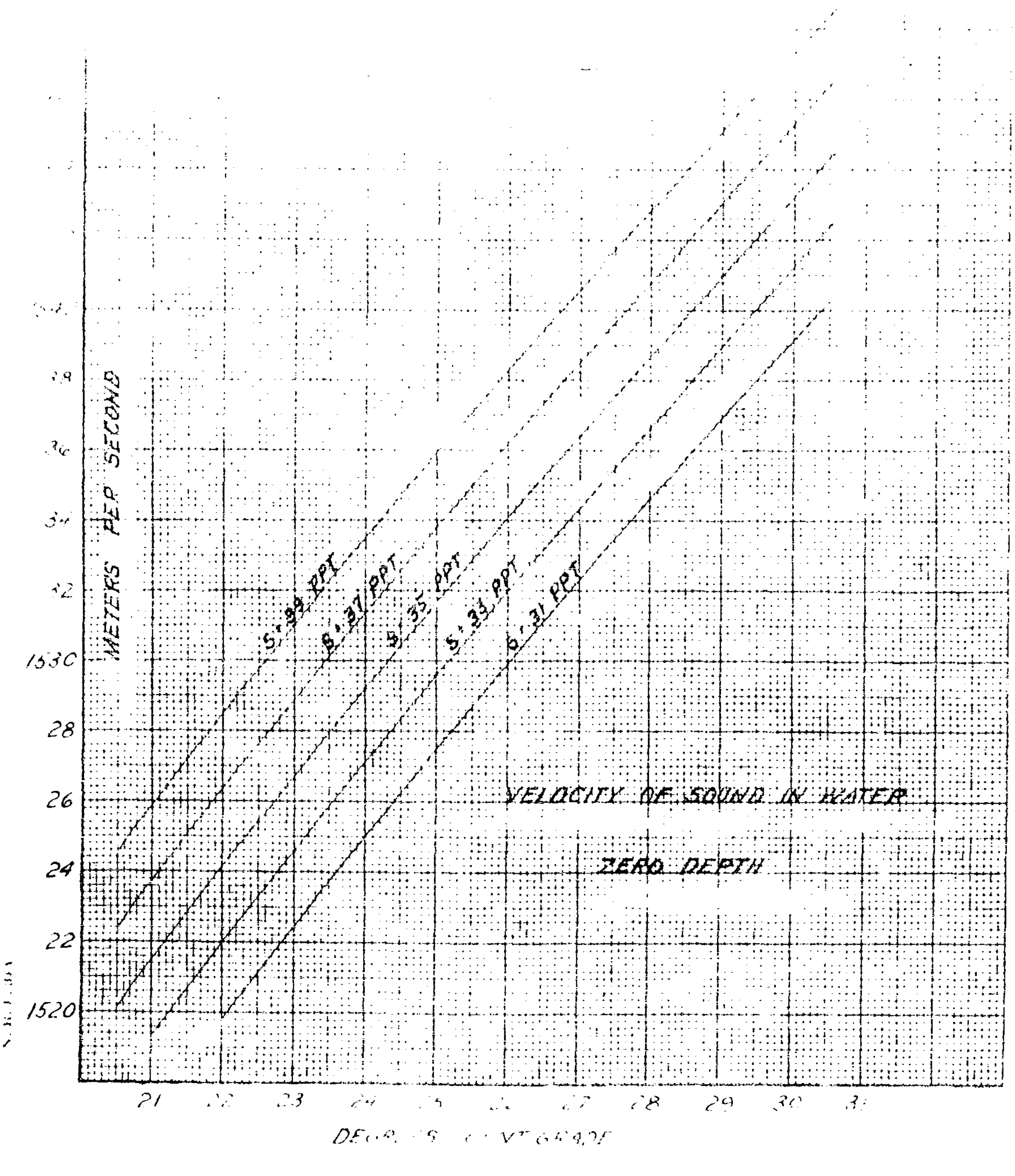


METERS PER SECOND
1480
1470
S: 32 PPT
S: 31 PPT
S: 35 PPT
S: 33 PPT
S: 31 PPT
VELOCITY OF SOUND IN WATER
ZERO DEPTH



VELOCITY OF SOUND IN WATER

ZERO DEPTH



METERS PER SECOND
1530
28
26
24
22
1520
21
23
28
29
S‰ 39 PPT
S‰ 37 PPT
S‰ 35 PPT
S‰ 33 PPT
S‰ 31 PPT
VELOCITY OF SOUND IN WATER
ZERO DEPTH

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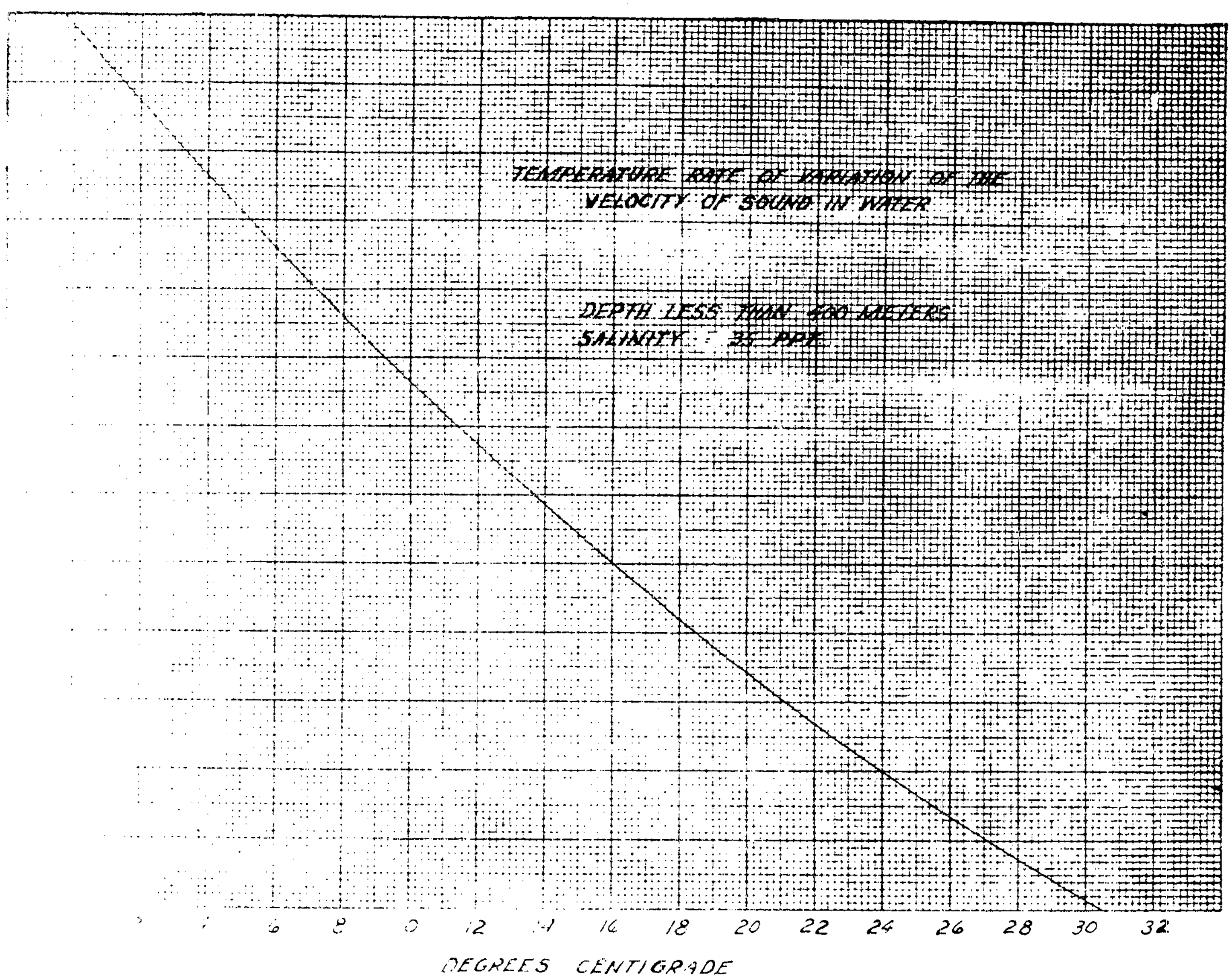

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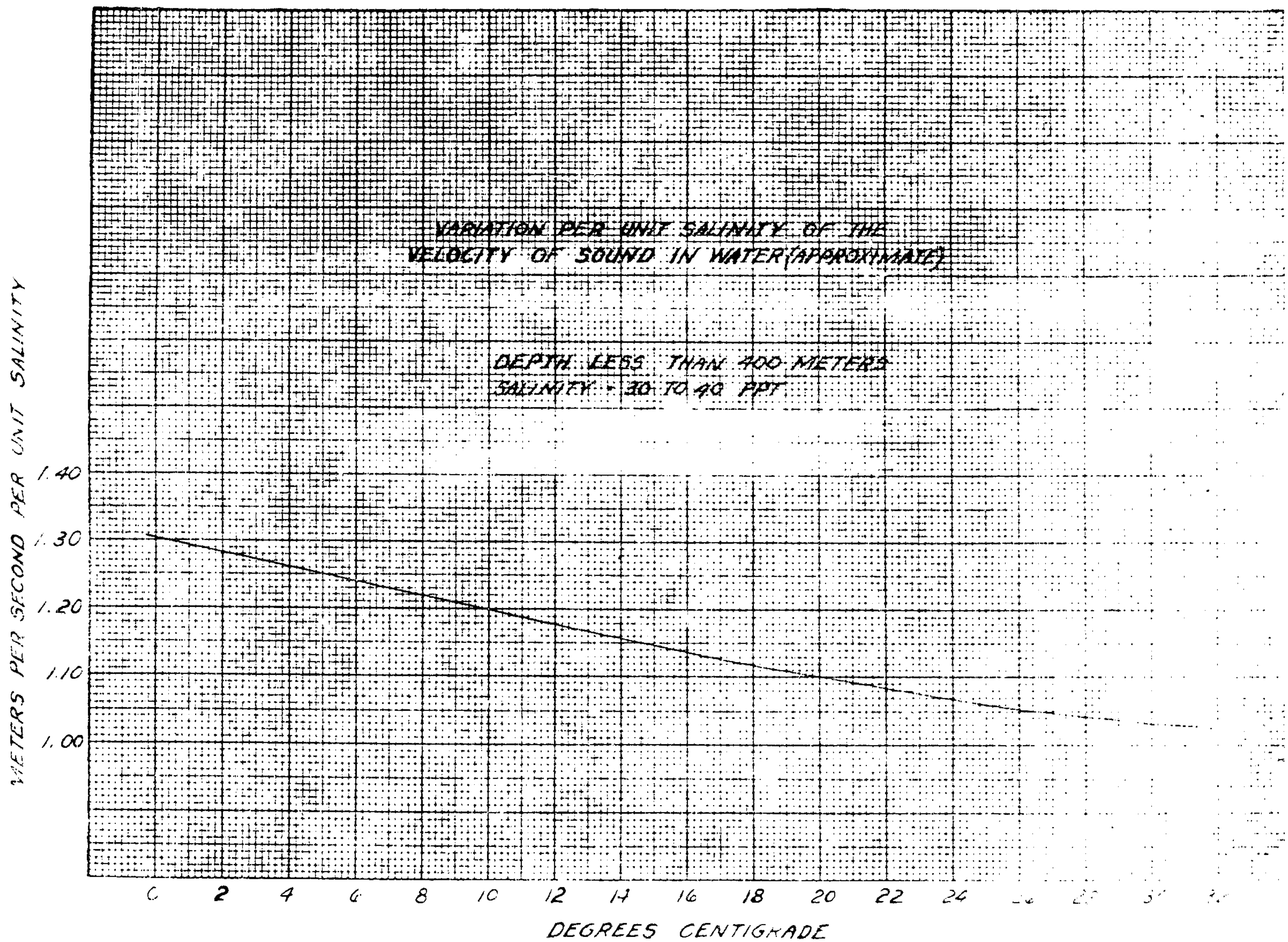
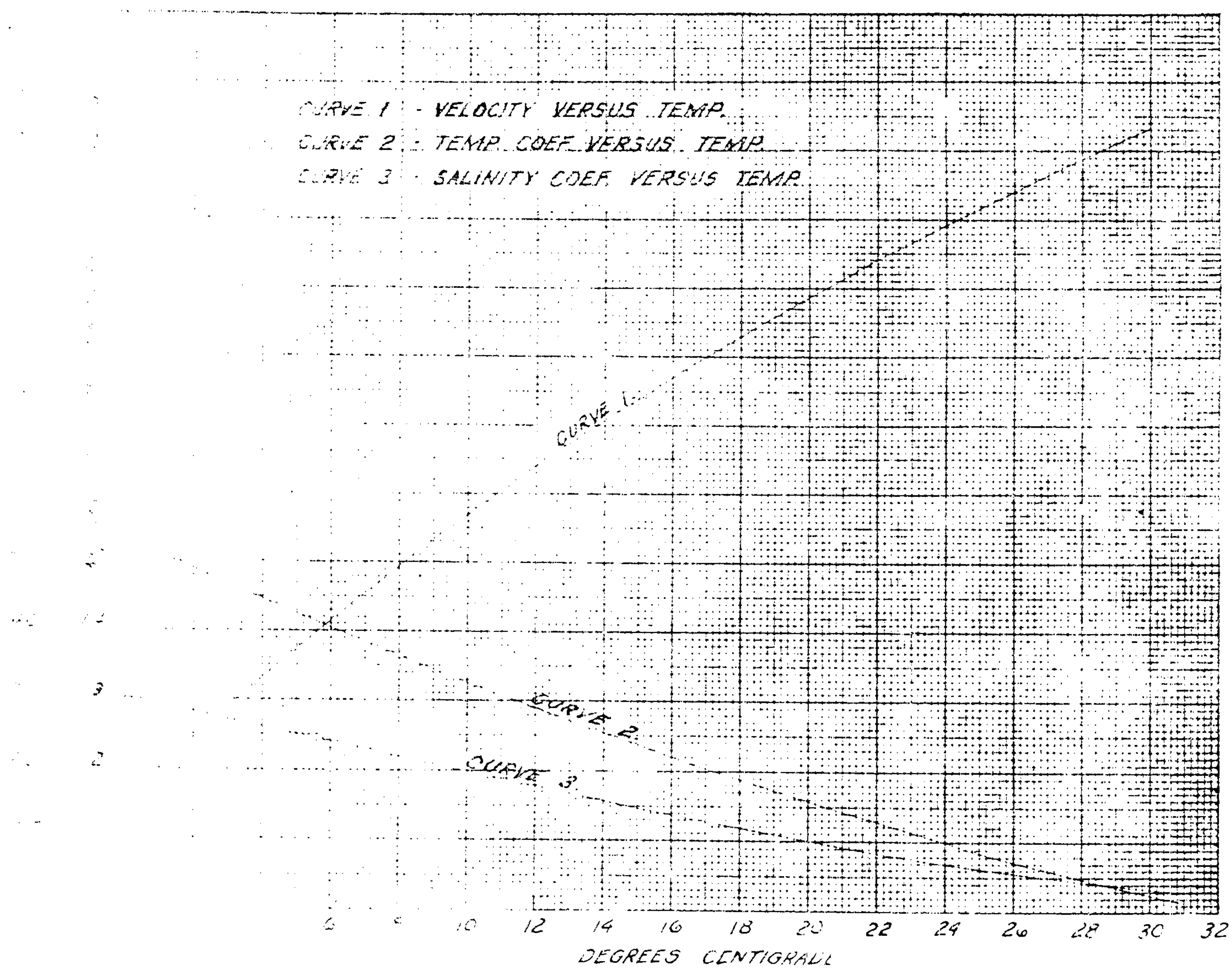


PLATE



CURVE 1 - VELOCITY VERSUS TEMP.
CURVE 2 - TEMP. COEF. VERSUS TEMP.
CURVE 3 - SALINITY COEF. VERSUS TEMP.
CURVE 1
CURVE 2
CURVE 3
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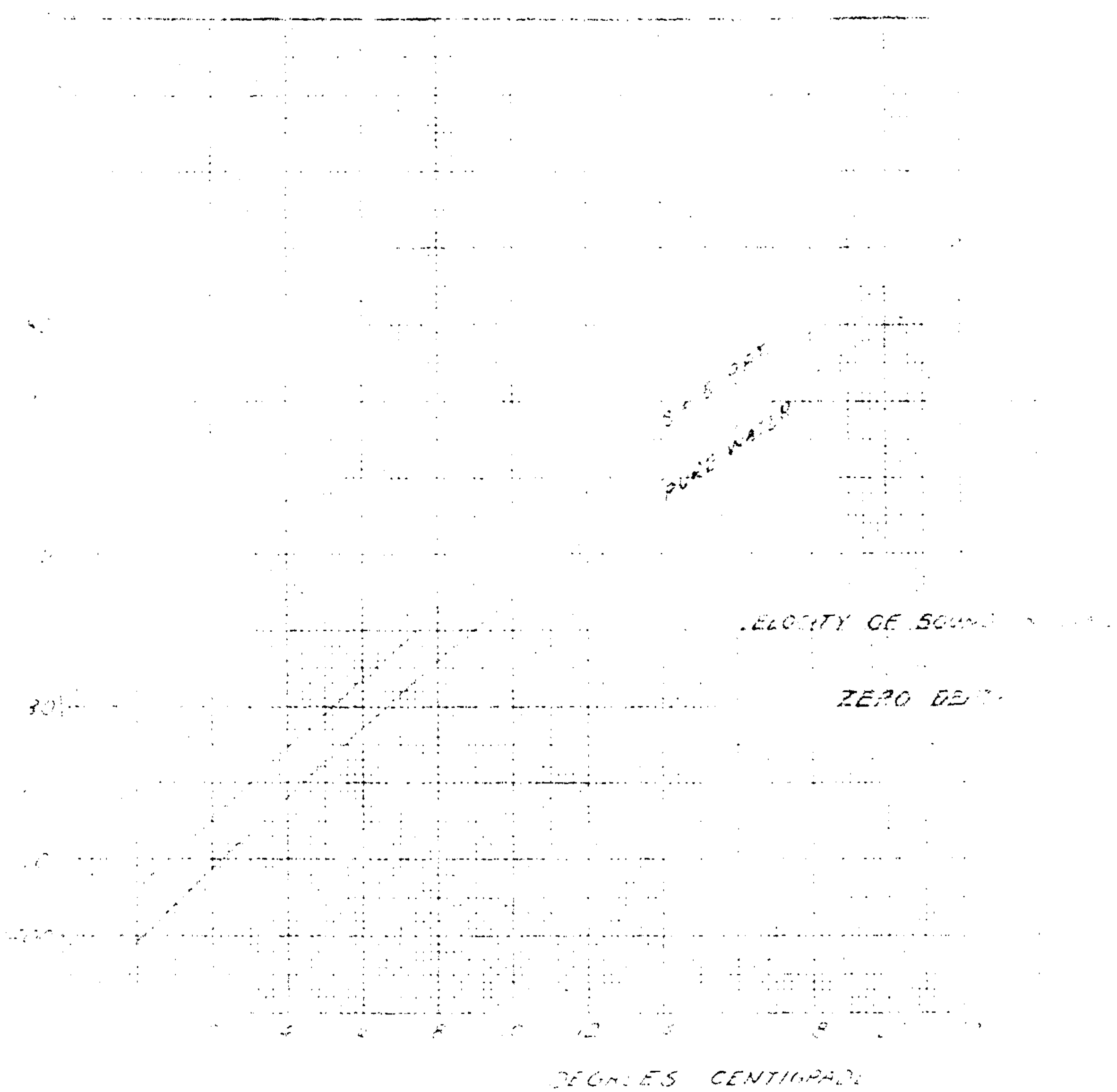
DEGREES CENTIGRADE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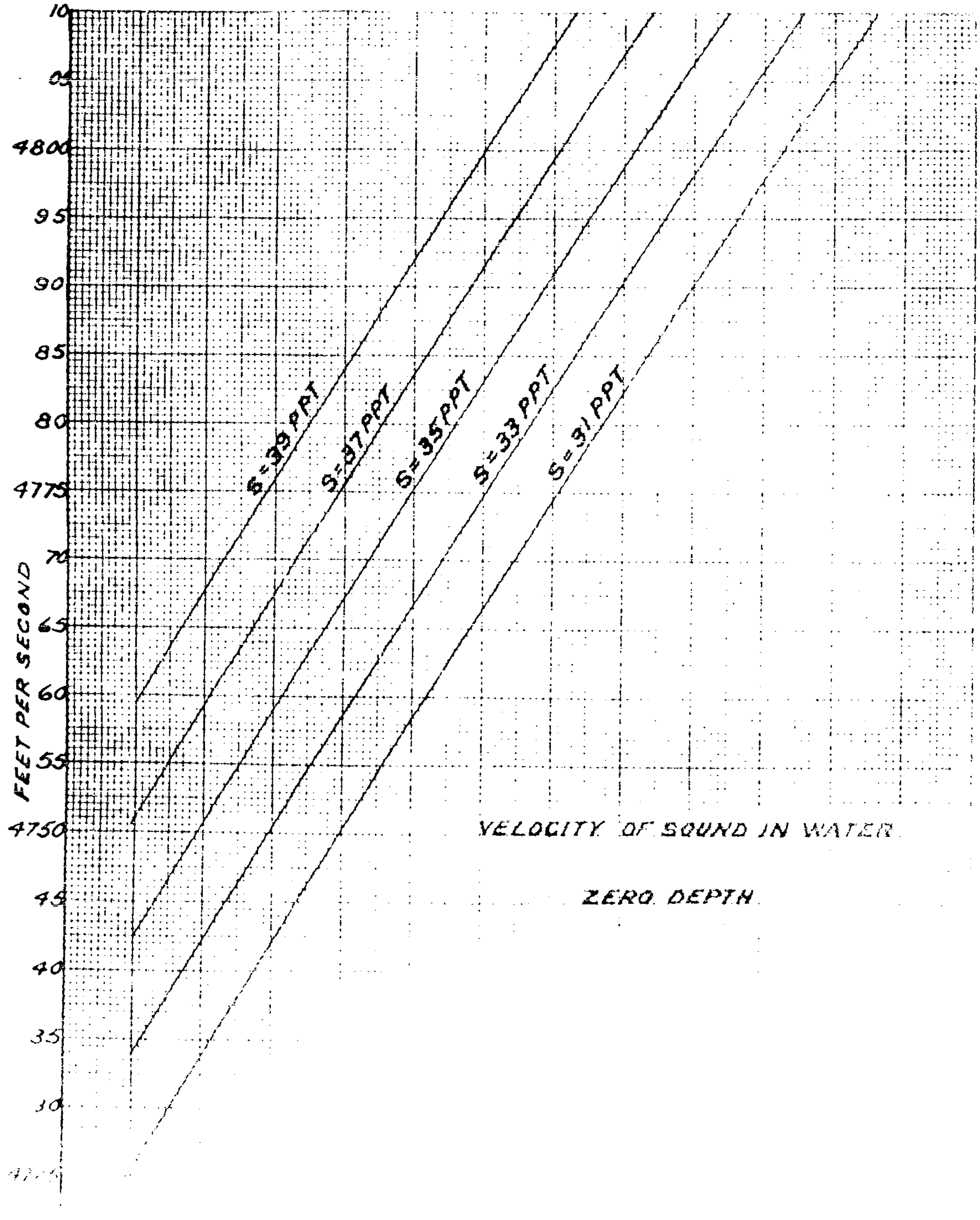
10
05
4800
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90
85
80
4775
70
65
60
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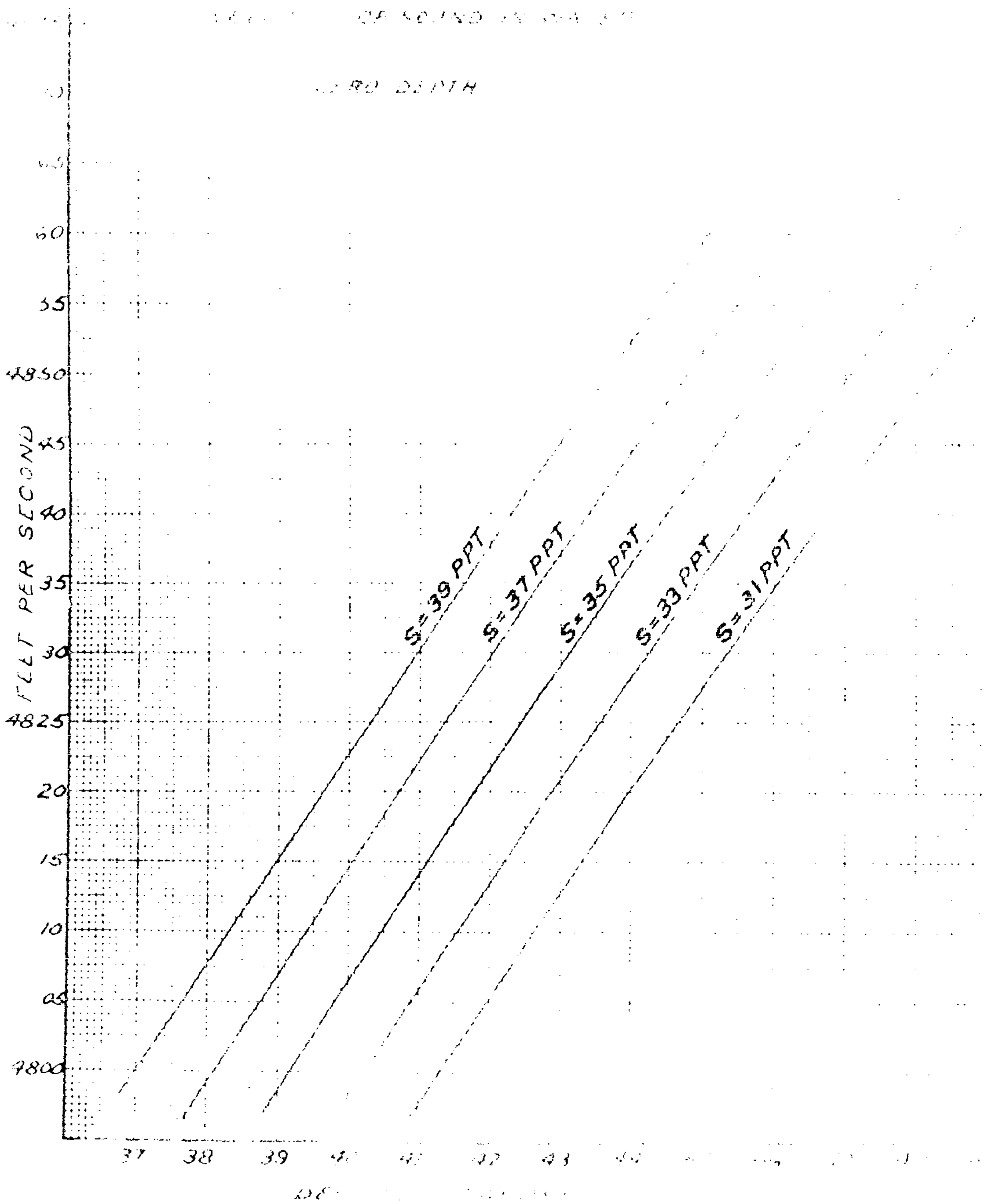
4750
45
40
35
30
FEET PER SECOND
S=39 PPT
S=37 PPT
S=35 PPT
S=33 PPT
S=31 PPT
VELOCITY OF SOUND IN WATER
ZERO DEPTH



OF SOUND IN SEA
RD DEPTH
FEET PER SECOND
4850
4825
4800
60
55
45
40
35
30
20
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05
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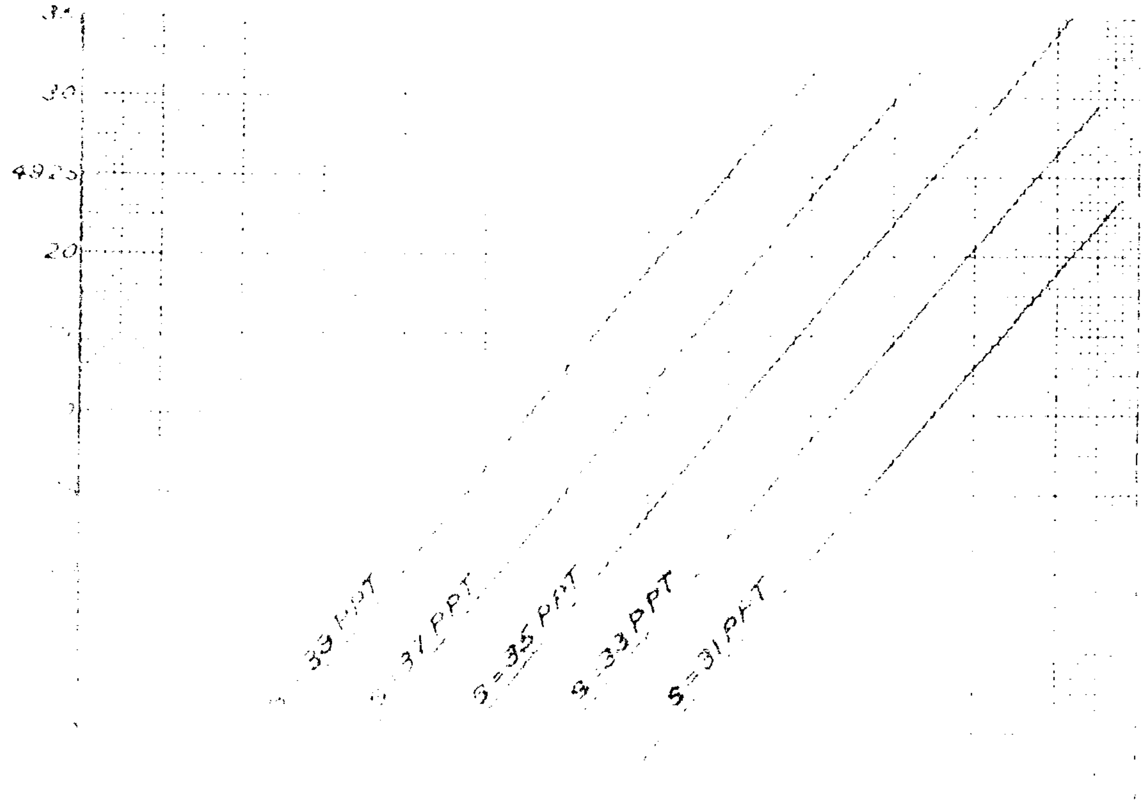
38
39
S=39 PPT
S=37 PPT
S=35 PPT
S=33 PPT
S=31 PPT



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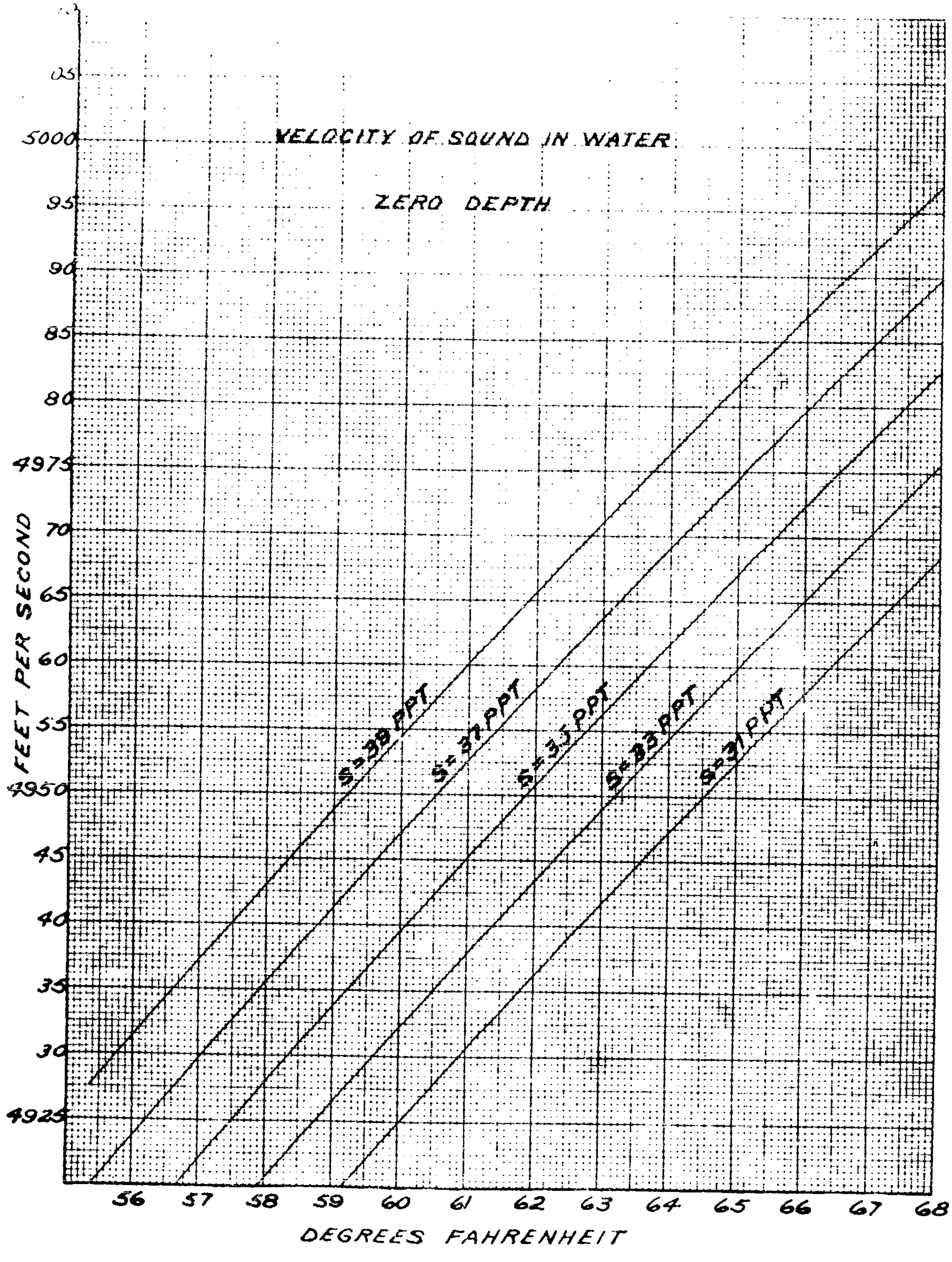
S=35 PPT
S=31 PPT


VELOCITY OF SOUND IN WATER
ZERO DEPTH
FEET PER SECOND
05
5000
95
90
85
80
4975
70
65
60
55
4950
45
40
35
30
4925
S=39 PPT
S=37 PPT
S=35 PPT
S=33 PPT
S=31 PPT
56
57
58
59
60
61
62
63
64
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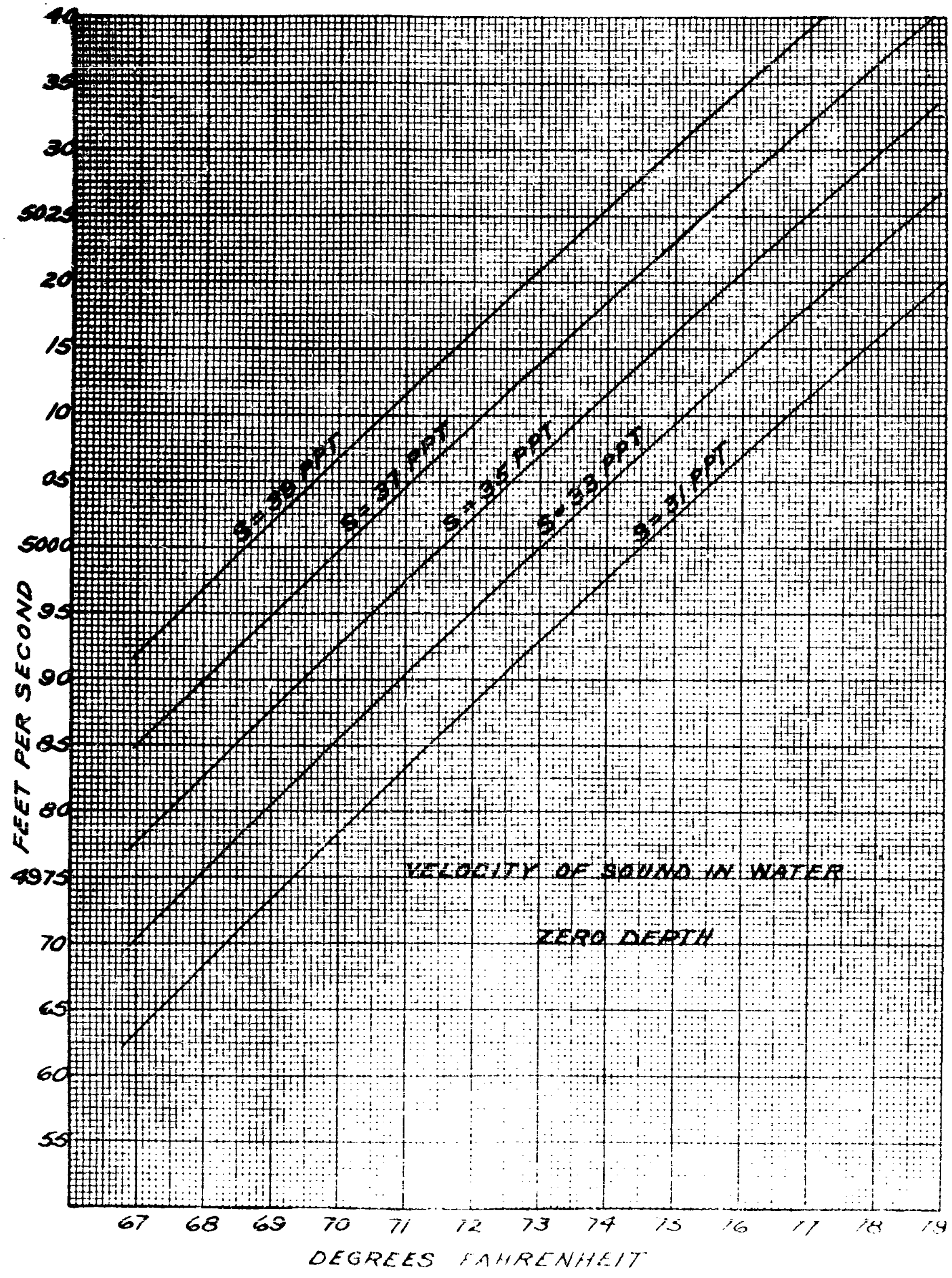
66
67
68
DEGREES FAHRENHEIT

PLATE 6-D

U. S. NAVAL RESEARCH LABORATORY
BELLEVUE, D. C.

DATE



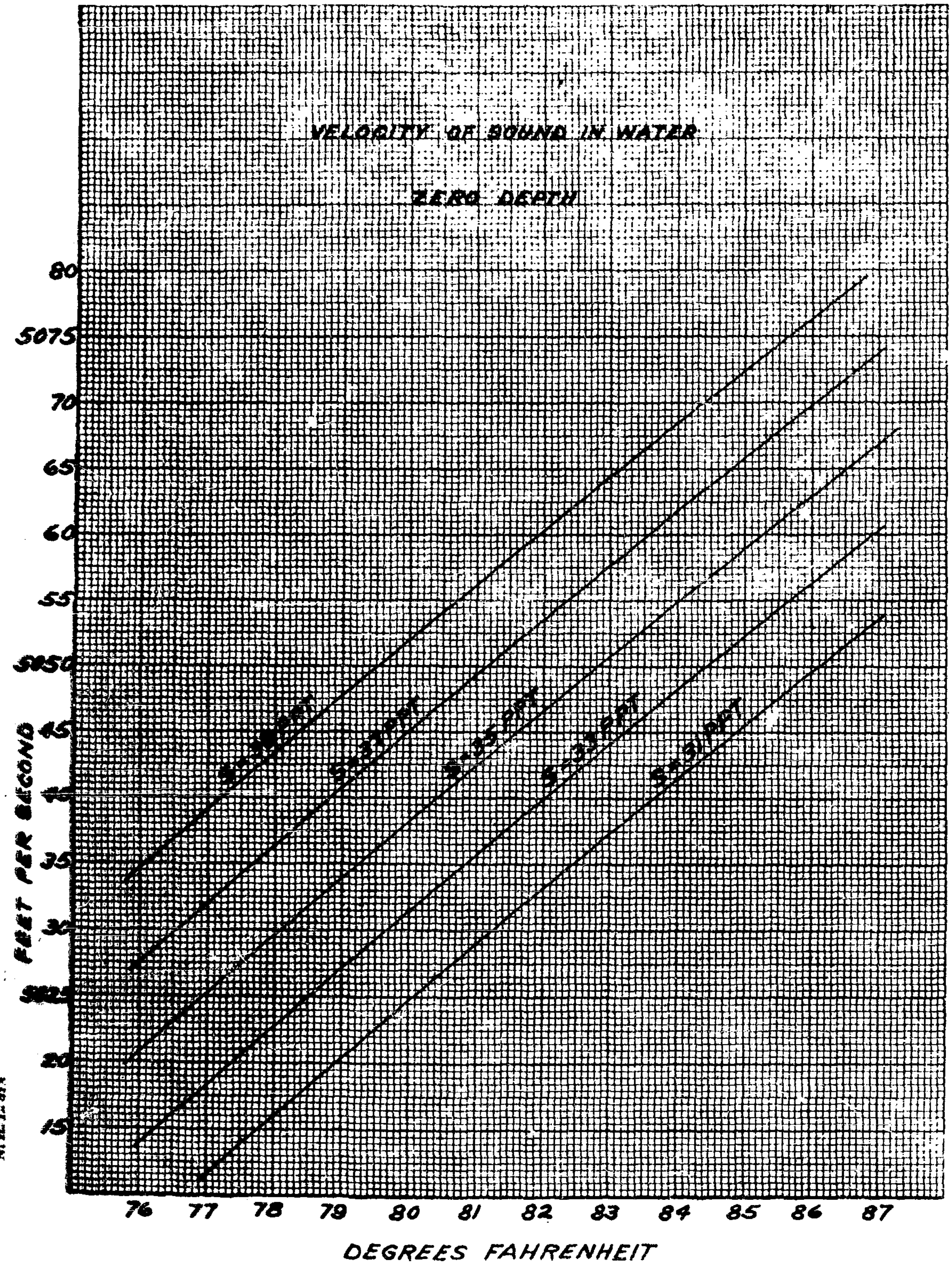
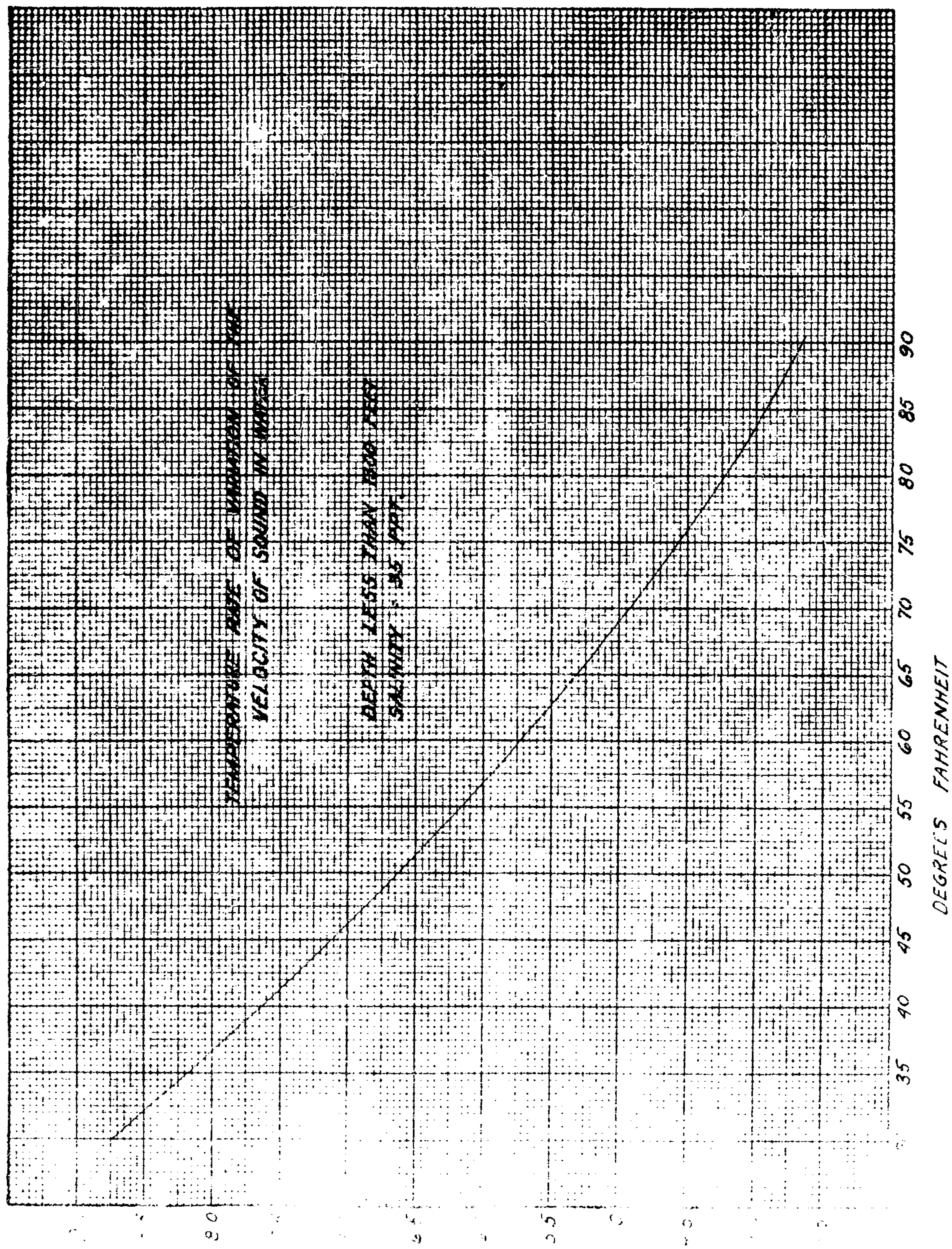

VELOCITY OF SOUND IN WATER
ZERO DEPTH
80
5075
70
65
60
55
5050
45
40
35
30
5025
20
15
FEET PER SECOND
S=37PPT
S=35PPT
S=33PPT
S=31PPT
76
77
78
79
80
81
82
83
84
85
86
87
DEGREES FAHRENHEIT

PLATE 6-F

U. S. NAVAL RESEARCH LAB
BELLEVUE, D. C.

DATE



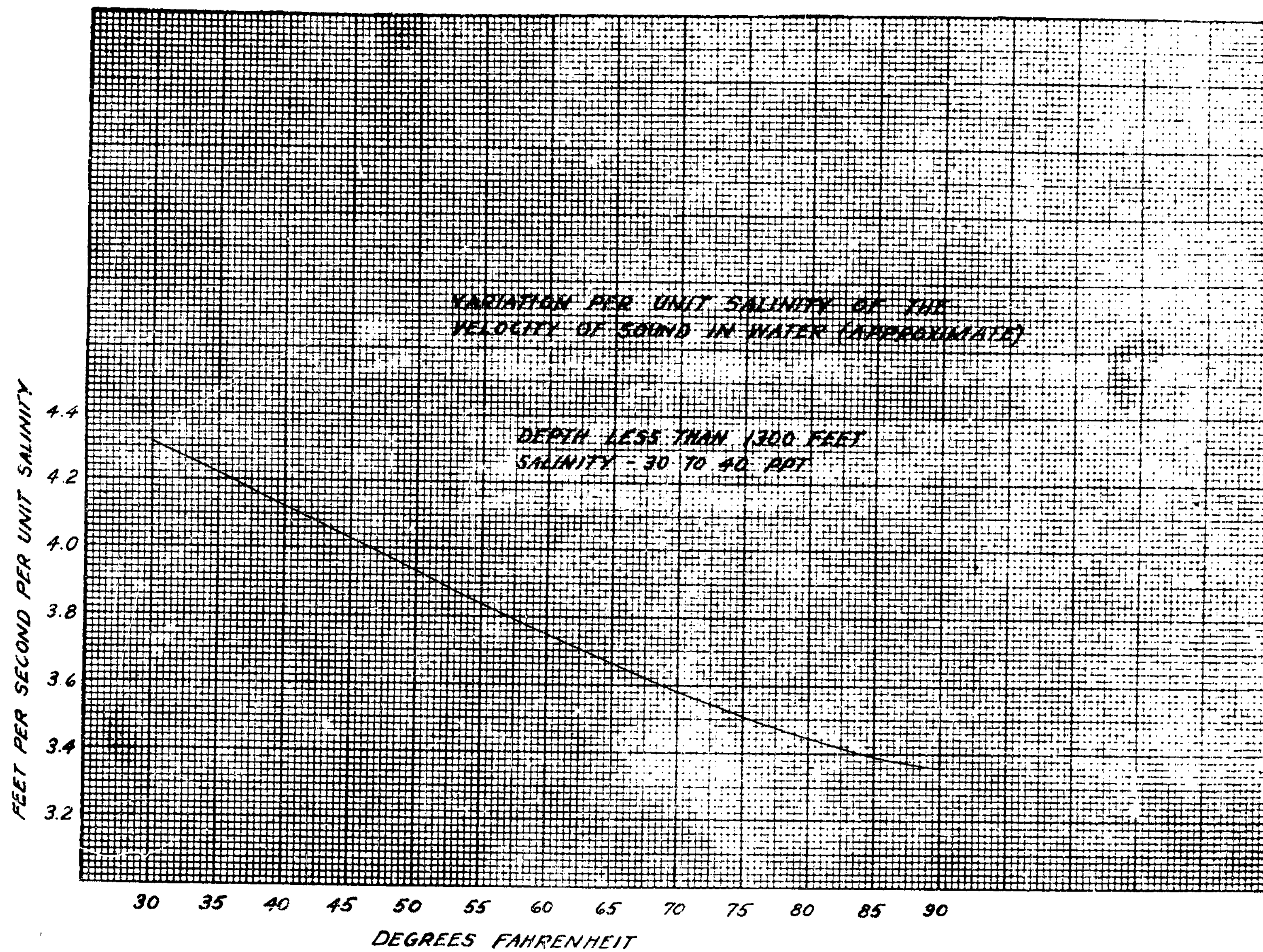
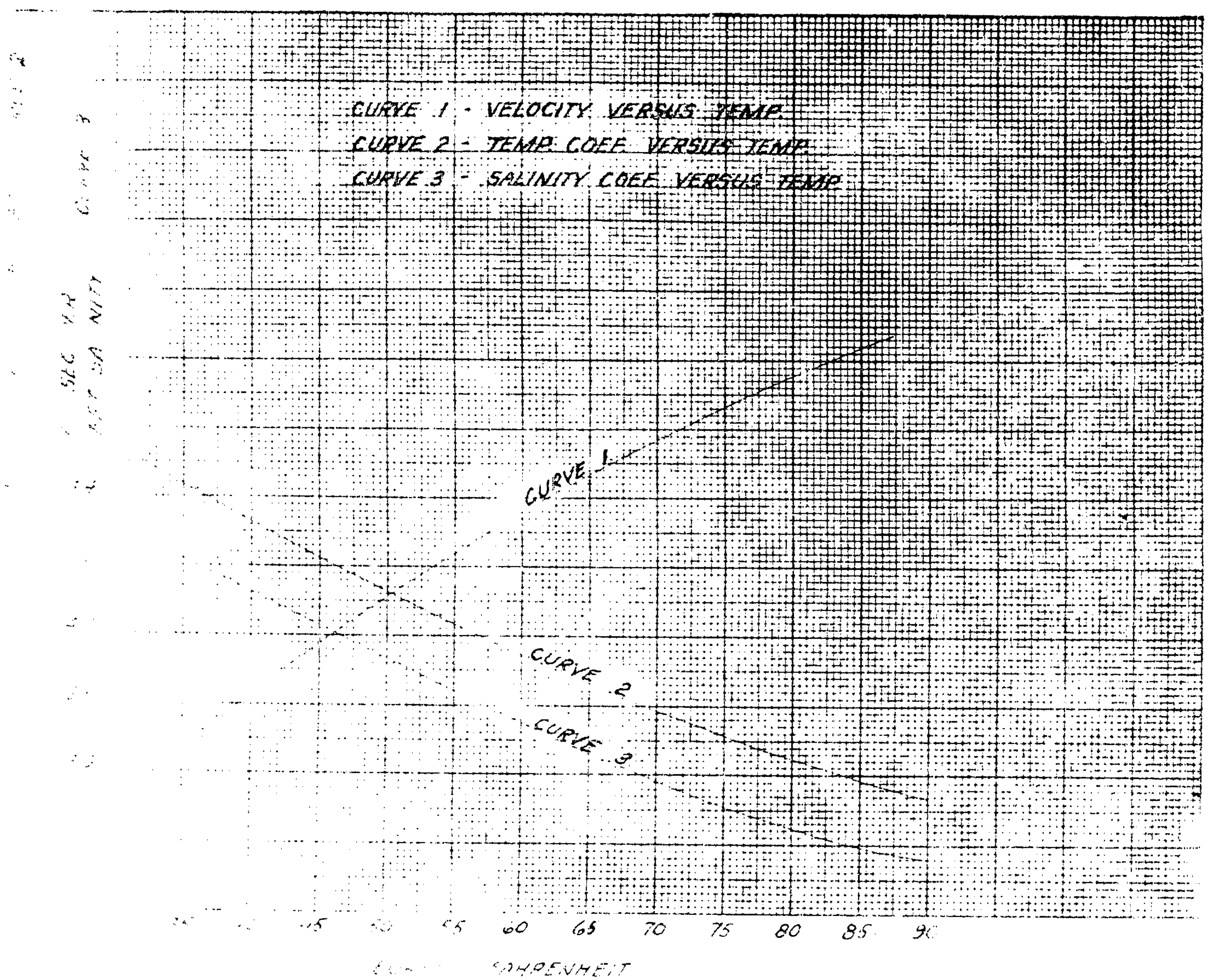
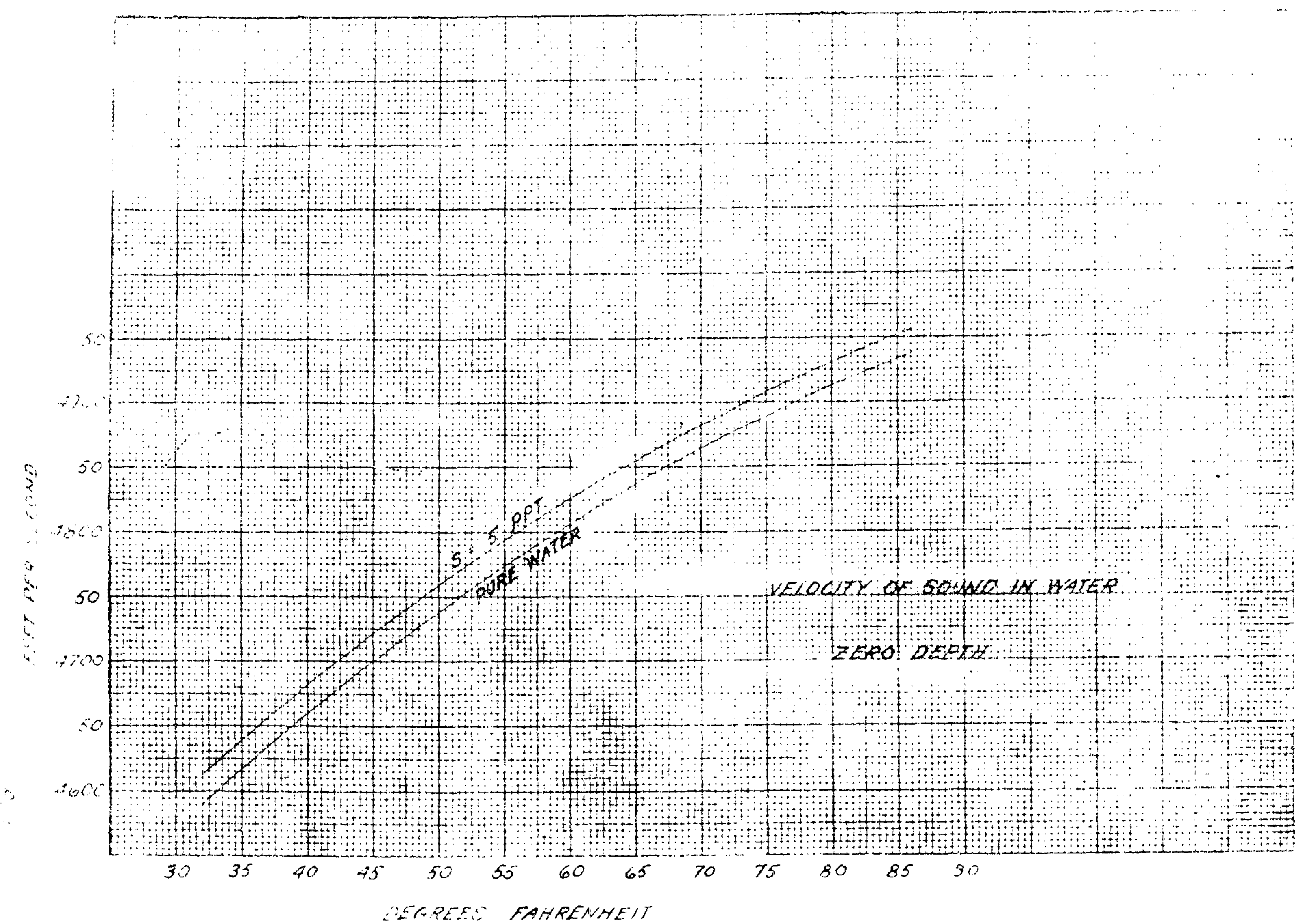

VARIATION PER UNIT SALINITY OF THE
VELOCITY OF SOUND IN WATER (APPROXIMATE)
DEPTH LESS THAN 1300 FEET
SALINITY - 30 TO 40 PPT
FEET PER SECOND PER UNIT SALINITY
4.4
4.2
4.0
3.8
3.6
3.4
3.2
30
35
40
45
50
55
60
65
70
75
80
85
90
DEGREES FAHRENHEIT

PLATE 8

S MUST BE TOP. IF SHEET IS READ THE OTHER WAY (VERTICALLY) THIS MUST BE LEFT-HAND SIDE.




VELOCITY OF SOUND IN WATER
ZERO DEPTH
5.5 PPT.
PURE WATER
4600
50
4700
50
4800
50
50
30
35
40
45
50
55
60
65
70
75
80
85
90
DEGREES FAHRENHEIT

UNITED STATES GOVERNMENT

memorandum

7103/911

DATE: 25 August 1999

FROM: Burton G. Hurdle (Code 7103)

SUBJECT: REVIEW OF REFS. (a) THROUGH (c) FOR REMOVAL OF RESTRICTIONS

TO: Code 1221.1 8/27/99

VIA: Code 7100

REF: (a) NRL Report S-1204, 16 Oct 1935, E.B. Stephenson AD-491 574
(b) NRL Report S-1670, 3 Dec 1940, E.B. Stephenson AD-135 780
(c) NRL Report S-1722, 11 April 1941, E.B. Stephenson and F.J. Woodsmall AD 221 613

1. References (a) through (c) are a series of reports and documents in underwater acoustics. Refs. (a-c) have been declassified earlier, but restrictions still exist.

2. The science, technology, equipment and operational utility of these reports have long been superseded. The current value of these reports is historical.

3. Based on the above, it is recommended that references (a) through (c) be available with no restrictions.

Burton G. Hurdle
BURTON G. HURDLE
Acoustics Division

CONCUR:

Edward R. Franchi 8/26/99
EDWARD R. FRANCHI Date
Superintendent
Acoustics Division

OPTIONAL FORM NO. 19
(REV). 1-80)
GSA FPMR(41 CFR) 101-11.6
5010-114